REVIEW.[1]

RESULTATE DES INTERNATIONALEN BREITENDIENSTES. Vol. I
(1903), by TH. ALBRECHT. Vol. II (1906), by TH.
ALBRECHT and B. WANACH. Centralbureau der Interna-
tionalen Erdmessung; neue Folge der Veröffentlichungen,
Nos. 8 und 13.

In the latter part of 1899 six astronomical stations were
established in the northern hemisphere for the purpose of
making systematic observations for the latitude of each station
in order to determine the variations in this quantity. Observa-
tions at the six stations have been carried on continuously since
they were established without serious interruption from any
cause, and are to be continued into the future for an indefinite
period of time. This work is being prosecuted by the Inter-
national Geodetic Association, which has headquarters at
Potsdam, Germany. This association was formed for the pur-
pose of conducting geodetic undertakings which are inter-
national in character. It is supported by the most prominent
nations of the world, including nearly all the governments
of Europe, the United States, the Argentine Republic, and
Japan. The geodetic institutes of the various countries in
which the latitude stations are located co-operate with the
central station in carrying on this particular piece of work,
the three stations in the United States being under the super-
vision of the Coast and Geodetic Survey.

Thus far the International Geodetic Association has pub-
lished two volumes (under the title at the head of this review)
giving the results of the observations for the variation of
latitude. Volume I contains all of the observations made

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[1] The writer of this review has endeavored to make it both popular and tech-
nical. It is hoped that the general reader may be able to get from it a general
knowledge of the problems presented under the head of the variation of latitude
and the methods by which they are attacked. It is hoped also that the profes-
sional astronomer or the student of astronomy who cares to look into the subject
more carefully may be able to get from it a fair idea of the details of the
processes involved. No attempt has been made by the reviewer to explain in de-
tail the parts which are purely technical and not essential to a general understand-
ing of the problems involved. In a number of places the language of the authors
has been followed quite closely, but in no case is the translation literal enough
to warrant the use of quotation-marks. For some of the opinions expressed, and
for Figure 1, and for the computations under the theory of probabilities, the
authors are in no wise responsible. In order to protect them, some of the state-
ments, with which the reader might not agree, and for which the reveiwer alone
is responsible, are followed by the letter r in parenthesis.

from the time the stations were established until January 4, 1902, and Volume II the observations obtained in the interval from January 5, 1902, to January 4, 1905. A third volume is in the course of preparation.

The contents of Volume I are presented under eight headings, which are indicated by italics in the following paragraphs.

Introduction.—The phenomenon of the variation of latitude was first detected in 1889 by Dr. KÜSTNER, astronomer in the Royal Observatory at Berlin. Various observations and investigations during the first half of the last decade of the nineteenth century established the reality and the nature of the phenomenon, and steps toward a systematic and thorough attack upon the problems presented were first taken by the International Geodetic Association in 1895.

In selecting the stations, social, hygienic, seismological, and meteorological, as well as mathematical, conditions were considered, the prime requisite being, of course, that all of the stations should have a fair proportion of clear nights at all seasons of the year. Seventeen different combinations of stations lying between latitudes $+ 36°$ 48' and $+ 44°$ 50', and including two combinations in the southern hemisphere on parallels $— 33°$ 54' and $— 33°$ 27', were considered. The parallel of $+ 39°$ 8' was finally chosen with the stations located in Japan, in Italy, and in the eastern and western parts of North America. Two other stations were subsequently added, one in Central Asia and the other in the central part of North America, at Cincinnati.

For the four stations first established—Mizusawa, Carloforte, Gaithersburg, and Ukiah—four new instruments exactly alike were constructed by WANSCHAFF in Berlin, 108^{mm} aperture, 130^{cm} focal length, 104 magnification. The instruments at Tschardjui and Cincinnati, by the same maker, are smaller, 68^{mm} aperture, 87^{cm} focus, and 81^{mm} aperture, 100^{cm} focus, respectively, both having 100 magnifying power.

The Horrebow-Talcott method[1] of observation was selected as the best suited for the purpose of determining the latitude.

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[1] Descriptions of this method may be found in any work on practical or general astronomy. The following statements concerning the method may be of help to those who are not familiar with its details. In order to make a determination of the latitude by this method it is necessary to measure, by means of an eye-piece micrometer attached to the zenith-telescope, the *difference* of zenith-distance of two stars of known declination which culminate at nearly equal zenith-distances, one north of and the other south of the zenith. The telescope is set at the mean

Twelve groups of stars, each containing six pairs at small zenith-distances (not more than 24°) and two pairs at large zenith-distance (about 60°), were selected. The stars were chosen by Dr. KIMURA, astronomer in charge of the Japanese station at Mizusawa. The magnitudes of the stars lie between 4.0 and 7.4, and the intervals between their culminations vary between four and sixteen minutes.

Two groups extending over four hours are observed each night according to the following programme:—

Group.	R. A.	To be Observed From	To		Duration of Group.	Group Connection.
I	0ʰ – 2ʰ	Sept. 23 – Dec.	6		74 days	35 days
II	2 – 4	Nov. 2 – Jan.	4		64	29
III	4 – 6	Dec. 7 – Jan.	30		55	26
IV	6 – 8	Jan. 5 – Feb.	24		51	25
V	8 – 10	Jan. 31 – Mar.	21		50	25
VI	10 – 12	Feb. 25 – Apr.	15		50	25
VII	12 – 14	Mar. 22 – May	11		51	26
VIII	14 – 16	Apr. 16 – June	8		54	28
IX	16 – 18	May 12 – July	9		59	31
X	18 – 20	June 9 – Aug.	13		66	35
XI	20 – 22	July 10 – Sept.	22		75	40
XII	22 – 24	Aug. 14 – Nov.	1		80	40

of the zenith-distances of the two stars and the first to culminate will pass a little above or below the middle of the field of view. This distance from the middle of the field is measured by means of the micrometer. The instrument is then reversed about its vertical axis, without disturbing the setting, and the telescope will then point as far south as it did north of the zenith before reversal, or *vice versa*. The second star will then pass through the field of view as far below or above as the first star was above or below the center, and this distance from the center is again measured by means of the micrometer. The proper combination of the micrometer settings on the two stars gives the actual difference of their zenith-distances, which may be turned into arc measure, provided the value of one revolution of the micrometer-screw be known. The latitude, ϕ, of the place of observation is computed by means of the formula,

$$\phi = \tfrac{1}{2}\,(\delta_n + \delta_s) + \tfrac{1}{2}\,(m_n - m_s)\,R + \tfrac{1}{2}\,(l_n + l_s) + \tfrac{1}{2}\,(r_n - r_s),$$

in which the first term of the right-hand member of the equation represents one half the sum of the declinations of the two stars of the pair observed; the second term one half the difference of the zenith-distances of the two stars as measured by means of the micrometer; the third term a small correction for any change in the pointing of the telescope after reversal, detected by means of two very delicate levels attached to the telescope; and the last term a small correction for the *difference* in the atmospheric refraction affecting the rays of light coming from the two stars. It might be noticed that if the two stars are at *exactly* the same zenith-distance, and the instrument is reversed without disturbing the pointing, then the second, third, and fourth terms each become zero in the equation above, and the latitude is nothing other than the mean of the declinations of the two stars, or the declination of the zenith.

As two groups are observed each night, it is seen that each group will be observed both with the preceding and the following one, for lengths of time which vary between twenty-five and forty days. This interval is made to vary simply for the, convenience of the observer, for by this means the observing time is made to come earlier in the evening during the winter months than during the summer months. In winter the observations lie between 7 P.M. and 1 A.M., in summer between 9 P.M. and 3 A.M. The time of beginning is never less than one and one-half hours after sunset, and the time of ending never less than one and three-quarter hours before sunrise. As the heating effects before sunrise are less pronounced than the cooling effects after sunset, it might perhaps have been wise to shift the whole programme a little further into the night. (*r*)

The observatory buildings at the six stations are of similar construction but differ somewhat in details. The one at Ukiah is three meters square, built of wood, with tin roof, and surrounded by an open slatwork construction which serves in some measure to protect the building within from the fierce rays of the summer sun. The roof is divided in the meridian-line and mounted upon rollers so that the two halves may be rolled apart, one to the east, the other to the west, giving a maximum opening of 1.8 meters. A small house to protect the meridian targets is located fifty-five meters north of the telescope.

Description of the Stations.—Detailed descriptions of the six stations and their surroundings are given, covering eight pages of the quarto volume. A few of the chief facts only will be mentioned here.

The city of Mizusawa (10,000 inhabitants) is situated on the principal Japanese island (Nippon), 466 kilometers north of Tokyo. The city lies in a north and south valley 180 kilometers long and five to fifteen kilometers wide. There are ranges of mountains to the east and to the west of the valley, the highest peak having an altitude of 2,200 meters. The valley is given largely to the cultivation of rice. The observatory is located about one kilometer south of the city. The number of earthquakes at Mizusawa is large, but the locality is not affected by these disturbances as much as some other portions of Japan. The zenith-telescope at this station was injured during the transport from Potsdam and the observations obtained with it

during the first year were subject to rather large errors. There are two observers at Mizusawa, Dr. H. KIMURA and Dr. T. NAKANO, who have served continuously since the observatory was established.

Tschardjui is located east of the southern end of the Caspian Sea in the Central Asian possessions of the Russian Government. The station lies nine and one-half kilometers northwest of the city and three kilometers from the left bank of the Amu Daria or river Oxus. The observatory is located on an oasis in a sand-waste traversed by many canals. There is a greater range in the annual temperature at this station than at any of the others. The early observations at Tschardjui did not show a satisfactory agreement among themselves. This was found to be due to a poor level and the use of oil illumination. Electric illumination was substituted and the level discarded. It might not be out of place to remark, parenthetically, that it is now generally admitted that the heat from oil-lamps may have a very injurious effect upon observations in which a high degree of precision is expected. Since a satisfactory electric illumination for intermittent work may be obtained by the use of any good make of ordinary dry cells, there seems to be no longer any excuse for using oil illumination for work with a zenith-telescope or altazimuth. (*r*)

Tschardjui is affected by very few earthquakes. The observations at this station are made by a single observer. Several have thus far taken part, and they have all been officers of the Russian army.

The Italian station has a very picturesque location on an old tower, San Vittorio, on the island of San Pietro, one kilometer southwest of the city of Carloforte. The tower is located on a peninsula on the east side of the island, so that the meridian of the observatory lies entirely over the Mediterranean Sea, with the exception of 260 meters to the north and 220 meters to the south, and anomalies in the refraction would seem to be absolutely excluded. The island is free from mountains, the highest point being 211 meters above sea-level. The altitude of the observatory is twenty-two meters. Carloforte has 8,000 inhabitants and can be reached from Cagliari, the chief city of Sardinia, in eight hours. The island is free from earthquakes, there having been only four in nearly four hundred years of any considerable intensity, and none of these destruc-

tive. The observations at this station are made by two observers, who alternate with the nights. Several changes in the staff have taken place thus far, but all its members have been Italian astronomers.

The Gaithersburg Observatory is located one kilometer south of the village of that name, which is thirty-three kilometers northwest of the city of Washington. The observatory has an altitude of 165 meters above sea-level; the surrounding country is hilly. Mr. EDWIN SMITH, of the Coast and Geodetic Survey, made the observations at this station during the first year; Dr. HERMAN S. DAVIS during the succeeding five years. The work is now in charge of Dr. FRANK E. ROSS.

After the parallel of 39° 8′ had been selected for the location of the latitude stations it was found that this parallel passed through the grounds of the observatory of the University of Cincinnati, and Professor J. G. PORTER, director of the observatory, volunteered to carry on observations if he were provided with an instrument. The observatory is located upon a hill, twenty meters higher than the surrounding country, eight kilometers northeast of the city, and two kilometers east of the Ohio River. The altitude of the observatory is 247 meters above sea-level. Thus far all of the observations, except a few during the summer months, have been made by Professor PORTER.

The California station is situated two kilometers south of the city of Ukiah, the county seat of Mendocino County. The observatory is located toward the western edge of one of the numerous small valleys in the Coast Range of mountains. The valley, which is traversed by the Russian River, is about fifteen kilometers long and from three to five kilometers wide, and surrounded by mountains of an average height of about 400 meters above the floor of the valley. The altitude of the observatory is 220 meters above sea-level. Up to May, 1903, the observations at this station were made by Dr. FRANK SCHLESINGER, now director of the Allegheny Observatory; since that time the work has been in charge of the writer of this review.

From a seismological point of view all of the American stations are favorably located. Although the Pacific Coast is well recognized as a region of seismic activity, yet the mountainous character of the country surrounding Ukiah seems to

afford a measure of protection from these disturbances. No earthquake since the observatory was established, not even the great shock of April 18, 1906, has been of sufficient intensity to in any way interfere with the progress of observations.

Instrumental Constants.—The most important constants to be determined in the case of a zenith-telescope are the angular value of one revolution of the micrometer-screw and the angular value of one space of the levels.

The pairs of stars have been so chosen that an error in the value of a revolution of the micrometer-screw will be eliminated from the mean of the latitude derived from each group. If the mean of the declinations of the stars of a pair, $\frac{1}{2}$ ($\delta_n + \delta_s$), is less than the latitude it must be increased by half the difference of the zenith-distances as measured by means of the microm- eter. If $\frac{1}{2}$ ($\delta_n + \delta_s$) is greater than the latitude, then the micrometer correction is to be applied with a negative sign. If now the value of a revolution of the micrometer-screw used is too small, all of the micrometer corrections will be *numer- ically* too small, and hence latitudes in the first case above will all be too small, and in the second case all too large. Hence, if in any group the sum of the positive micrometer corrections is made equal to the sum of the negative corrections, the errors will be eliminated in the mean latitude as determined from that group. On account of the precession this elimination will hold only for a certain epoch, and the same group cannot therefore be used for an indefinite period. In the work of the International Geodetic Association the first selection of groups was used for six years, three years on either side of the epoch 1903.0.

The angular value of one revolution of the micrometer- screws was determined by the use of two methods, transits of polar stars at elongation, and measurement of differences of declination of stars as they come to the meridian. The second of these methods was used only at Cincinnati and Ukiah. The chief objection to it is that the results are affected by what- ever errors may pertain to the declinations of the stars used. The first method, transits of polar stars at elongation, is theo- retically preferable, but in practice gives results which show rather a large and unsatisfactory range. This is very likely due to the fact that the observer must assume either that the angle between the line of sight of the telescope and the vertical

remains unchanged during the progress of the observations, an hour, more or less, or that any changes in this angle are truthfully indicated by the readings of the latitude levels. Levels at their best are untrustworthy instruments, and in this case, since it is necessary for the observer to stand during the whole progress of the observations near the south end of the level tubes, it is easily conceivable that the heat from the observer's body may so affect the levels that a change in the reading of the bubbles may take place without any corresponding change in the pointing of the telescope, or *vice versa.* (*r*)

The value of one revolution of the micrometer-screw depends of course upon the temperature at which the determination is made. The range in temperatures at Carloforte and Ukiah is not sufficient to enable a good determination of the temperature coefficient to be made, and the observed values at the other stations depart rather widely from the theoretical values computed from the coefficients of expansion of brass and steel and the known values of one revolution of the screws. These differences are to be explained through the statement that the temperature coefficient evidently depends upon factors other than those just stated.

All of the screws were investigated for both progressive and periodic errors, either by observation of polar stars at elongation or by the use of auxiliary apparatus. Periodic errors of a sensible magnitude were found only for the instruments at Tschardjui and Carloforte. The screw of the Cincinnati telescope was found to be practically free from both progressive and periodic errors. The progressive errors of the screws at Carloforte and Gaithersburg were found to be considerably larger than the values obtained at Potsdam in tests applied before the instruments were shipped. The only explanation seems to be that the screws were damaged in some way during transportation.

The values of the spaces of the latitude levels were determined by micrometric settings upon the mire or by the use of a level-trier. At Ukiah a new method of observing stars of nearly the same declination was tried by Dr. SCHLESINGER with good success. This method has certain decided advantages and the details of it have already been explained in these *Publications* (Vol. XIII, p. 13). The effect of temperature and barometric pressure on the levels was not investigated.

Instrumental Errors.—Before making observations for lati-
tude with a zenith-telescope it is necessary to so adjust the
instrument that any instrumental errors which may remain shall
be so small that they will not have an appreciable effect upon
the accuracy of the results to be obtained. In order to attain
this end the vertical axis must be made truly vertical, or very
nearly so, the horizontal axis truly horizontal and in the plane
of the prime vertical and the collimation zero, or rather of the
same magnitude as the flexure of the horizontal axis, in order
that the one may counteract the other, or that the collimation
minus the flexure may be nearly zero. The position of the axes
may be tested by means of the levels attached to the instru-
ment and by the mire. The collimation, flexure, and position
of the meridian targets can be tested only by observations of
the stars for time. Since the telescope in this type of instru-
ment is attached to one end of the horizontal axis and a
counterpoise of equal weight at the other end, the flexure of
the horizontal axis is large, about two seconds of time, and
a time determination with this instrument involves a laborious
process unless the flexure be assumed as a known quantity.

Volumes I and II give the daily values of the instrumental
errors for each station.

Atmospheric Conditions.—Observations of the inside and
the outside temperature and of the barometric pressure are
made hourly during the progress of observations. The only
use to be made of these would be in the investigation of pos-
sible cases of abnormal refraction. It is the *difference* of the
refraction of the two stars of a pair which enters into the
computation of the latitude, and under normal conditions this
may be computed for stars at small zenith-distances by means
of a formula based upon a mean value of the temperature and
the barometric pressure.

In addition to the individual temperature and barometric
readings there is given a tabulation showing the mean tem-
perature for each group connection at each station. The great-
est range, $35°.6$ centigrade, is at Tschardjui, the least at
Carloforte and Ukiah, $13°.7$ and $14°.8$ respectively. The range
in group-connection means is, from $-8°$ to $+21°$ at Mizu-
sawa, from $-9°$ to $+26°$ at Tschardjui, from $+10°$ to
$+23°$ at Carloforte, from $-5°$ to $+23°$ at Gaithersburg,
from $-4°$ to $+24°$ at Cincinnati, from $+4°$ to $+19°$ at

Ukiah. At the last-named place, although the midday temperature often reaches 40° (104° Fahrenheit), and at times has reached as high as 45° (113° Fahrenheit), yet the temperature decreases rapidly immediately after sundown, and such a thing as a hot night is practically unknown.

Results of Observations.—The individual values of the latitude from each pair observed, computed by means of the equation already given, are found in tabular form in this section. The total number of determinations made during the period covered by the first volume is 27,387. The percentage of nights upon which observations were obtained at the various stations is as follows:—

Mizusawa	48 per cent.
Tschardjui	33 "
Carloforte	70 "
Gaithersburg	48 "
Cincinnati	34 "
Ukiah	47 "
Average	47 per cent.

The conditions at Carloforte, in the Mediterranean Sea, must be almost ideal from an astronomical standpoint, still the above tabulation cannot be taken as a true index of the weather at the stations. At Carloforte and at Mizusawa two observers are constantly employed, and probably nearly every favorable night is utilized. At the other stations, where all of the observations are made by a single observer, many favorable nights must of necessity be allowed to pass. At Ukiah, for instance, the percentage could be increased by at least ten, perhaps fifteen, if two observers were employed. In considering the above table the further fact should be taken into consideration that Professor PORTER, who makes the observations at Cincinnati, has many other duties in connection with his position as director of the Cincinnati Observatory and professor of astronomy in the University of Cincinnati. With him observing for latitude is an avocation and not a vocation. We should also consider the still further fact that at some stations,—for instance, Mizusawa,—many nights are rendered incomplete by fog or clouds, and a night upon which only one pair is obtained enters into the above tabulation with the same weight as a complete night of sixteen pairs. During the 750 days since observations

were begun at the last station to start, Mizusawa, there were
only five upon which observations were obtained at all six
stations.

Determination of the Definitive Latitudes.—The first inves-
tigation undertaken under this heading was to determine
whether or not there was a systematic difference between
observations taken east-west and those taken west-east. None
of significance was found except at Mizusawa, in November
and December, 1900. Only the seventy-two latitude pairs
were used in the definitive determination of the latitude.

The next step was to determine corrections to the declina-
tions of the stars from the observations themselves. There
are two steps in this process,—first, to determine the reduc-
tions to be applied to the results of each pair in order to reduce
them to a mean declination system of the group; second, the
determination of the reductions necessary to bring the groups
to a common basis. The first step was accomplished by taking
the mean of the results from the six zenith pairs on each
night that the complete group was observed, and then sub-
tracting the individual results, including those for the two
refraction pairs, from this mean. The results for each pair at
each station were then collected and the grand means taken
as the corrections to $\frac{1}{2} (\delta_n + \delta_s)$ for each pair.

The total mean error of the results will be made up of the
accidental errors of observation and the systematic station
errors, which last are either instrumental or personal, or per-
haps of an external nature, such as anomalous refraction. The
numerical value of the first of these, the accidental error of
observation, may, on account of the richness of the observa-
tional material, be computed from the individual observations
of a pair, thus eliminating all consideration of the errors of
declination. The mean error for a single observation of a
pair is about $\pm 0''.15$. The total mean error of the station
mean for a single pair is $\pm 0''.067$ for the latitude pairs, and
$\pm 0''.090$ for the refraction pairs. These values are greater
than those to be obtained from the accidental errors alone,
showing that there are in fact systematic station errors. It
is not possible as yet to state what these are due to, but an
examination of the ten pairs having the largest differences of
zenith-distance seems to show that they may lie in errors
in the assumed values of the micrometer-screws.

The corrections to the group means were applied to the mean group latitudes and the results assembled on pages 121-125. From these, differences between the group results for each group connection were obtained for each station and the results collected. After the weighted means for the six stations had been taken, it was found that their algebraic sum differed appreciably from zero. One explanation of this, and perhaps the most probable, is that the value of the constant of aberration used in the star-place reductions was in error. It was found that a correction to the aberration of $+ 0''.042$ would cause the "Schlussfehler"—closing error—to disappear. This would change the adopted constant of aberration from $20''.470$ to $20''.512$, a change which is confirmed by other investigations entirely separate from this. It might be stated here that the computations given in volume II cast some doubt upon the reality of this correction.

From the group differences the corrections to each group were formed which are necessary to reduce the twelve groups to a common declination system based on the declinations of the seventy-two latitude pairs. From these reductions to a mean system and the reductions to the group means the final corrections to the declinations of each pair were formed and collected into a table (page 129). These corrections were then applied to the daily means at each station, and these then formed into the group combination means, the latitude pairs and the refraction pairs being treated separately.

Determination of the Path of the Pole.—The path of the pole was determined from the normal values of the latitude for each group connection. The first step in this process was to determine the mean value of the latitude for each station, and this was done by a method of successive approximations. The differences between the mean values and the individual mean group connection values were then formed and plotted with time as the x co-ordinate. A mean curve was drawn for each station and then the values of $\phi - \phi_0$ (ϕ_0 being the mean latitude of the station) were read from it for each tenth of a year, thus eliminating in a large measure the errors of observation.

It was first assumed that the motion of the pole could be represented by an equation of the form,

$$\Delta \phi = x \cos \lambda + y \sin \lambda,$$

in which $\Delta \phi = \phi - \phi_0$, λ the longitude of the place of observation west of Greenwich, x and y the rectangular co-ordinates of the instantaneous pole in a system the origin of which is located at the position of the mean pole, the x axis of which points toward Greenwich, and the y axis toward a point in 90° west longitude. The above equation may be derived easily from the following figure.
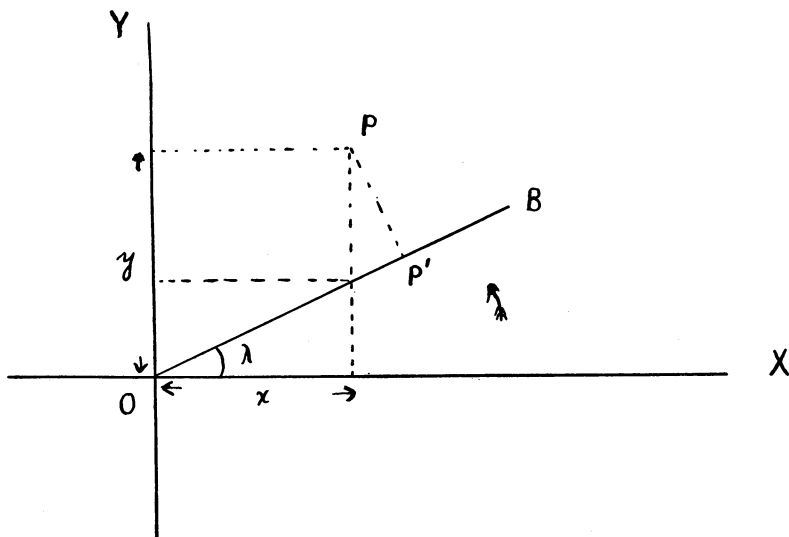

FIG. I.

Let $O B$ be the direction toward any observatory in west longitude λ, P the instantaneous position of the pole, and P' the foot of the perpendicular dropped from P upon $O B$. Then we have,

$$O P' = \Delta \phi = x \sec \lambda + (y - x \tan \lambda) \sin \lambda,$$
$$= x \left(\sec \lambda - \frac{\sin^2 \lambda}{\cos \lambda} \right) + y \sin \lambda,$$
$$= x \cos \lambda + y \sin \lambda.$$

Early investigations showed that the observations were not satisfied very well by this equation, and Dr. KIMURA suggested the introduction of a third unknown independent of the longitude, thus, $\qquad \Delta \phi = x \cos \lambda + y \sin \lambda + z.$

Least-square solutions under both assumptions were made, the one involving the solution of six equations for two unknowns

at each tenth of a year, the other the solution of six equations for three unknowns at each tenth of a year. The sum of the weighted squares of the residuals is so measurably better under the second assumption that there seems to be no doubt about the existence of the term z. The largest residuals were found in the case of Gaithersburg, and they were so much larger than the others that the case seemed to demand special investigation. The period under discussion involved a change of observers at Gaithersburg, SMITH leaving at the close of 1900. The two observers, however, did not observe together long enough to determine their personal equation. It was found that the large residuals at Gaithersburg could be greatly reduced by *assuming* a personal equation of a tenth of a second,— namely, ϕ Davis — ϕ Smith = $+ 0''.10$. Upon the introduction of corrections based upon this assumption the residuals for Gaithersburg, and also for Cincinnati, were reduced to the same order of magnitude as those of the other stations. The difference between the results obtained by two observers lies perhaps not so much in the bisections of the stars as in the general handling of the instrument, especially, in the opinion of the reviewer, in the manipulation of the levels.

The final values of x and y give the motion of the pole as represented in the first part of the curve of Figure II, reprinted from these *Publications*,[1] and showing also the motion of the pole as obtained from subsequent observations up to the beginning of 1906.

It is not possible to decide from the data at hand whether or not the values of z derived for these stations hold also for other latitudes. The question can be most easily decided by establishing latitude stations in the southern hemisphere, and this has now been done. Three explanations are offered by Dr. ALBRECHT in regard to the term z—anomalous refraction, a north and south oscillation of the center of gravity of the Earth, the effect of the neglected annual parallax of the fixed stars. The values of z for the six different stations were found to be practically the same, so that the hypothesis of anomalous refraction seems to be excluded.

Volume II.—The second volume contains the presentation of all the observations made between 1902, January 5th, and 1905, January 4th. The treatment in this volume is in all

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[1] Vol. XVIII, No. 111, p. 315.

respects similar to that given in Volume I, except that the
section on the description of the observatories is omitted and
a section is added at the end under the heading, "Derivation
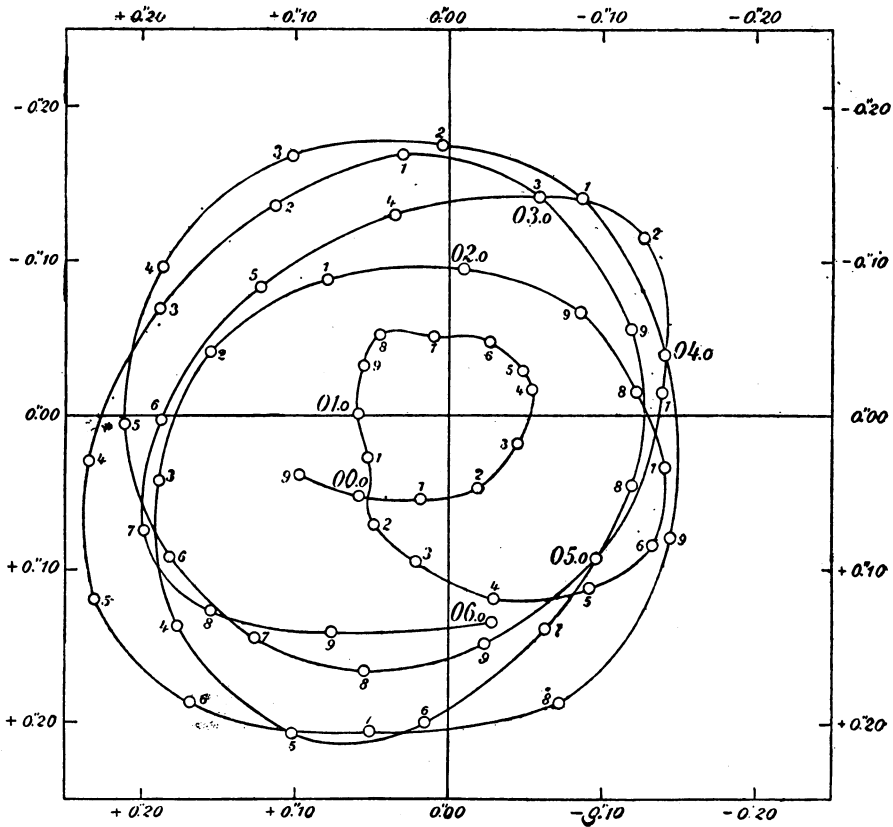and Discussion of the Results of the Refraction Pairs."   It

FIG. II.

is intended to call attention here only to those points at which
something new is brought out in the second volume.

Additional observations were made at all of the stations for
values of one revolution of the micrometer-screws, and an at-
tempt was made to determine the temperature coefficients.   As
before stated, these were determined at all of the stations
except Carloforte and Ukiah, where the range of temperatures
is not sufficiently great to enable a good determination to be

made. Since all of the instruments were made by one maker, after the same pattern, differing only in size, it was decided to assume that the temperature coefficient was the same for all. From the observations for the four instruments for which determinations have been made, the weighted mean temperature coefficient, the change per degree centigrade per second of arc of the micrometer-screw, was found to be — 0.0000259, and the resulting values for the six screws are then easily found. Before the final results for Volume II were worked out, corrections to the observed values of one revolution of the micrometer-screws were determined from the latitude observations themselves. These corrections lie between — 0.0009, for Gaithersburg, and — 0.0251, for Tschardjui.

The total number of observations obtained during the period under consideration in the second volume is 36,173. The percentage of nights upon which observations were made is 46.5, almost exactly the same as during the period covered by Volume I. There was an increase in the percentage at four of the stations and a decrease at Gaithersburg and Cincinnati. The percentage at Carloforte increased from 70 to 73.

During the interval under consideration, 1,096 days, there were only fourteen evenings on which observations were obtained at all six stations, and on three of these only one pair was obtained at some stations. There was not a single evening on which a complete programme was obtained at all six stations, and by looking back into Volume I it is found that there has not been a* single night, from the time the last station started, 1899, December 16th, to 1905, January 4th, 1,846 nights, upon which a complete programme was obtained at all six stations.

This seems a little strange at first thought, but a simple computation according to the principles of probability will show that we are here dealing with a very rare event. Let us ask, first, What is the probability of obtaining at least some observations at each station on the same night? If we assume that observations are made on the average on fifty per cent of the nights, then the probability of obtaining observations at any one station on any particular night will be one half; and manifestly the probability of obtaining observations at two stations on the same night will be $\frac{1}{2} \times \frac{1}{2}$, or $\frac{1}{4}$, and the probability of obtaining observations at three stations on the

same night $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, and the probability of obtaining observations at six stations $(\frac{1}{2})^6 = \frac{1}{64}$. Observations would therefore be made at all six stations on the same night on an average of once in every sixty-four nights. Our assumption, however, that observations are made upon fifty per cent of the nights is somewhat in error, the true percentage being almost exactly 46.5. The probability of this event occurring would be therefore $(\frac{465}{1000})^6$ which equals $\frac{1}{99}$. This event would occur on an average therefore of once in every ninety-nine days, or nineteen times during the 1,846 days under consideration.[1] This result is in exact agreement with the observed number; there were five such events during the period covered by Volume I and fourteen during the period covered by Volume II. (*r*)

Let us now ask, What is the probability of obtaining a complete night's work at all six stations on any particular night? The ratio between the number of complete nights and the total of nights observed is not given in the volumes, but it is probably not far from one half. At Ukiah about sixty per cent of the nights upon which observations are made are complete, but the percentage is known to be less at some of the other stations. If now we assume that observations are made upon fifty per cent of the nights, and fifty per cent of these are complete, then the same kind of reasoning that was used before will bring us to the conclusion that the probability of the occurrence of the event under consideration is $(\frac{1}{64})^2 = \frac{1}{4096}$. That is to say, a complete night's work will be obtained at all six stations on an average, in round numbers, of once in every 4,000 nights, or once in about eleven years, so that it is not at all surprising that this rare event did not occur at all during the first five years of observations. (*r*)

The mean errors of a single determination of the latitude are practically identical for all stations except Carloforte, which seems to show that as accurate observations can be made with the small instruments at Tschardjui and Cincinnati as with the larger ones. The observations made at Carloforte stand in a class by themselves, as far as accidental errors go, these being distinctly less than at the other stations, probably largely

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[1] The exact method of computing this probability is, of course, to take the product of the six separate probabilities rather than the sixth power of the average probability. The result comes out sixteen, rather than nineteen.

due to the favorable meteorological conditions.   An examina-
tion of the curves represented in Figure 3 shows, however,
that notwithstanding the small accidental errors and the large
number of observations obtained at Carloforte the final results
for this station are not as accurate as for some of the other
stations,—Mizusawa, for example,—where the taking of ob-

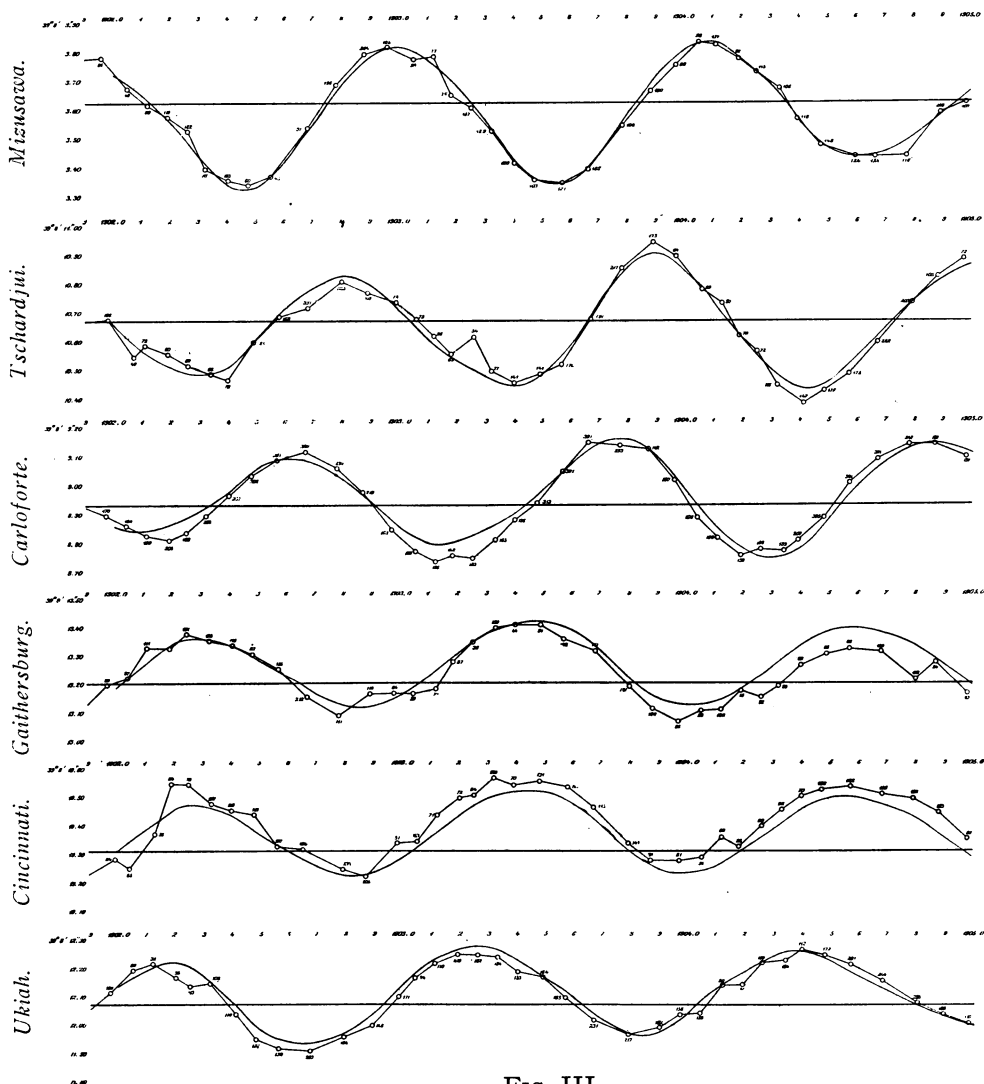*Verlauf der Polhöhe auf den einzelnen Stationen.*

FIG. III.

servations is often badly interfered with on account of clouds
or fog. Nearly twice as many observations are obtained at
Carloforte as at Mizusawa, and this is a good illustration of
the precept that little or nothing is to be gained by increasing
beyond a certain moderate amount the number of observa-
tions made with the same instrument under similar circum-
stances. Perhaps just as good results could be obtained by
limiting the number of observations at each station to 1,200
or 1,500 a year. (*r*)

In explanation of Figure III it should be stated that the
small circles represent the average observed latitude for cer-
tain intervals, the small numbers adjacent to the circles indi-
cating the number of observations entering into the average.
After the results were all combined and the most probable
values of x, y, and z were obtained from the least-square
solutions, as already explained, the values of x and y were
computed for each station at certain times and the results
plotted on the same figure with the observed curves, produc-
ing the smooth curves of Figure III. The amounts by which
the circles depart from the smooth curves, the residuals, are
probably very close to the true errors of observation, and
that station in which these residuals are the smallest has, of
course, procured the best results.

The "closing errors" vary so much more in the different
years for the same station than should be expected from the
accidental errors, that it seems necessary to conclude that
these differences cannot be all laid up to an erroneous value
of the aberration constant. It would be necessary to assume
the constant of aberration equal to $20''.541$ in order to com-
pletely explain away the closing errors. The differences
are perhaps due to meteorological causes,—a very handy
explanation.

The quantities z, as derived in the second volume, show a
tendency toward a change in a positive direction, which pre-
sumably is due to uncertainties concerning the knowledge of
the proper motions of the stars observed. An average change
of $+ 0''.016$ a year in the proper motions would cause the
positive tendency in z to disappear. The evidence seems to
show that we have in this quantity z a term of constant ampli-
tude and a period of a year. The observations thus far,
however, give no indication of the cause of this term.

Volume II is concluded with a discussion of the results obtained from the observations of pairs at large zenith-distances, the so-called refraction pairs. Several hypotheses concerning abnormal refractions were made, but no very definite conclusions concerning them could be drawn from the data at hand. Without taking up the details it is perhaps sufficient to state the general conclusion reached by the authors, and that is, that observations at 60° zenith-distance provide no evidence whatever from which conclusions can be drawn regarding refractive perturbations at small zenith-distances. The observation of refraction pairs was therefore discontinued at the beginning of 1906.

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